

Formation of on-site Normal Points

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Abstract.

We describe our methods of fitting a smoothing function to observational range differences from a predicted orbit, by deducing corrections to the orbit in the radial and along-track directions. The method has been used on observations of a variety of satellites, and using predicted orbits computed both by numerical integration using IRVs as starting values and analytically from orbital elements. The along-track corrections to the predicted orbit have been successfully used in the form of time biases to improve subsequent predictions, and a statistical test has been devised to ensure that the range residuals may be used to form unbiased quicklook normal points.

1. Introduction.

It was recommended at the Fifth International Laser Ranging Instrumentation Workshop held at Herstmonceux, UK, that laser range normal points should be formed by the stations shortly after each satellite pass and transmitted as quicklook data (Gaignebet, 1985). The final version of the quicklook normal point format was agreed by the SLR Subcommittee of the CSTG and published in the April 1990 Satellite Laser Ranging Newsletter (Schutz, 1990). The process of forming normal points consists of 2 stages; (a) fitting a smoothing function to the observational range differences from a predicted orbit and subtracting the function in order to form a flat track of residuals, and (b) rejecting noise and outliers and forming the mean values of the accepted residuals in bins spread through the pass. The values of the smoothing function and the predicted orbit at the epoch of an actual observation nearest the mean epoch of the bin are then added to the mean value of the residuals in the bin. The resulting normal points are then virtually independent of both the smoothing function and of the predicted orbit.

In this report we detail the processes adopted to carry out stage (a) of the process; methods for rejecting noise and forming unbiased normal point bin means are developed by Sinclair (these proceedings)

2. Development of the Smoothing Function.

During the observation of a satellite pass, differences from the predicted range are computed for all events detected within the range gate and displayed in realtime, true satellite returns appearing as a track of correlated values among the randomly distributed noise events. An example of such a plot is given in Figure 1, which shows a pass of ERS-1. The rejection of gross noise events is carried out visually using the plot, and the subsequent set of satellite

range measurements and some noise events are passed to the next stage of processing.

At an early stage of the development of the preprocessing software, the decision was taken to use the pass-by-pass observations to solve for corrections to the predicted orbit, and in particular to monitor the along-track error, in order to improve subsequent prediction accuracy. Thus the possible use of an arbitrary polynomial for the smoothing function was not considered. However, corrections to the predicted orbital elements of the satellite would be difficult to apply to orbits computed by numerical integration and using IRVs as starting values, and similarly corrections to the IRVs would not be applicable to orbits computed analytically. Thus it was decided to express orbital corrections as along-track, across-track and radial displacements to the predicted position. Such corrections are then readily applied to the geocentric orbit during the solution iteration process, and the derived along-track correction can be expressed as an orbital time-bias for improving subsequent predictions.

During the prediction process an orbit in the form of geocentric rectangular satellite coordinates and velocities at 1-minute time intervals and in units of Mm and Mm/day, is computed and stored. For each observational epoch, we use 8th order Lagrangean interpolation to compute the instantaneous position \mathbf{r} and velocity \mathbf{v} of the satellite. Let $\mathbf{r} = (x, y, z)$, and $\mathbf{v} = (v_x, v_y, v_z)$, and let the magnitudes of these vectors be r and v respectively. The reference frame used is different depending upon the source of the orbit being considered; for an orbit generated by numerical integration and using IRVs, the coordinates are given with respect to the true equator and mean equinox at 0hrs UT of the epoch of the IRVs. For an analytically-derived orbit, the coordinates are given with respect to the mean equator and equinox of date. At the observational epoch, we compute the coordinates of the observing station in the same reference frame. Let the station coordinates be $\mathbf{s} = (x_s, y_s, z_s)$. Then the predicted topocentric coordinates of the satellite are

$$x_T = x - x_s, \quad y_T = y - y_s, \quad z_T = z - z_s, \quad ,$$

and the predicted range is the magnitude of this topocentric vector. We thus form the difference of the observed from the predicted range, and in order to solve for corrections to the predicted orbit we require partial derivatives of range with respect to the along-track, across-track and radial components of the displacement of the predicted orbit.

2.2 Partial Derivatives.

(a) Let the along-track displacement or time-bias be ΔT .

Then the displaced range R is given by

$$R^2 = (x + v_x \Delta T - x_s)^2 + (y + v_y \Delta T - y_s)^2 + (z + v_z \Delta T - z_s)^2$$

Then

$$R \frac{\partial R}{\partial \Delta T} = v_x(x + v_x \Delta T - x_s) + v_y(y + v_y \Delta T - y_s) + v_z(z + v_z \Delta T - z_s),$$

or approximately

$$\frac{\partial R}{\partial \Delta T} = (v_x x_T + v_y y_T + v_z z_T)/R$$

(b) Let the across-track displacement be ΔQ

The across-track direction is

$$\frac{\mathbf{r} \times \mathbf{v}}{\mathbf{r} \cdot \mathbf{v}} = (l, m, n) \quad , \text{ say.}$$

So,

$$l = (y v_z - z v_y)/rv$$

$$m = (z v_x - x v_z)/rv$$

$$n = (x v_y - y v_x)/rv$$

So the displaced range R is given by

$$R^2 = (x + l\Delta Q - x_s)^2 + (y + m\Delta Q - y_s)^2 + (z + n\Delta Q - z_s)^2$$

So,

$$R \frac{\partial R}{\partial \Delta Q} = l(x + l\Delta Q - x_s) + m(y + m\Delta Q - y_s) + n(z + n\Delta Q - z_s),$$

or approximately

$$\frac{\partial R}{\partial \Delta Q} = (l x_T + m y_T + n z_T)/R$$

(c) Let the radial displacement be Δr

So the displaced coordinates of the satellite are $x + x\Delta r/r$, etc.

So the displaced range R is given by

$$R^2 = (x + \frac{x\Delta r}{r} - x_s)^2 + (y + \frac{y\Delta r}{r} - y_s)^2 + (z + \frac{z\Delta r}{r} - z_s)^2$$

Then as before we calculate the approximate partial derivative

$$\frac{\partial R}{\partial \Delta r} = (x x_T + y y_T + z z_T)/(rR)$$

2.3 Solutions.

In practice it was found that the across-track correction was always highly correlated with the radial correction, and in many cases the solution was indeterminate. Hence both could not be solved-for, and so it was decided to suppress the solution for across-track, and solve for a radial correction only, which would thus absorb the across-track error. It was also found that simple constant along-track and radial corrections to the predicted orbit did not in general absorb all the error in the orbits, and that the set of parameters to be determined for a particular pass should be selected from along-track and radial displacements and their time rates of change and accelerations. We denote these parameters $T, \dot{T}, \ddot{T}, R, \dot{R}, \ddot{R}$. The partial derivatives of range with respect to the rates of change and accelerations of these parameters were formed from those of the constant terms by multiplication by t and t^2 , where time t is the epoch of each observation relative to the mid-time of the pass. Such a definition for the origin of t is optimum in reducing correlations between the unknowns. It was usually found necessary to solve for only 4 of the possible six parameters, but occasionally all six were required to remove all trends from the observational residuals.

Initially a scheme was devised to automatically check for very high (>0.999) correlations among the 4 unknowns $\dot{T}, \ddot{T}, \dot{R}, \ddot{R}$, and to suppress any one or two of them in order to obtain a determinate solution, consistent with obtaining a flat track of residuals. However experience showed that the values determined for these 4 parameters were always quite small, of the order of 0.1 ms/minute for \dot{T} and 1.0 cm/minute for \dot{R} , and of similar magnitude for the accelerations. We thus imposed a-priori standard errors of these magnitudes upon the 4 parameters, and allowed the program to solve for all six unknowns.

In order to iterate the solution, at each stage we replace the predicted coordinates of the satellite by the displaced coordinates as determined from the previous solution. So for example

$$x = x + v_x \Delta T + l \Delta Q + x \Delta r / r$$

and similarly for y and z . It was found that 4 or 5 iterations were usually sufficient, where outliers of magnitude greater than $3 \times \text{rms}$ were removed at each stage.

A selection of some plots of observational residuals is given in Figure 2, where it is clear that all trends have been removed; these residuals are readily used to form normal points using the Herstmonceux algorithm.

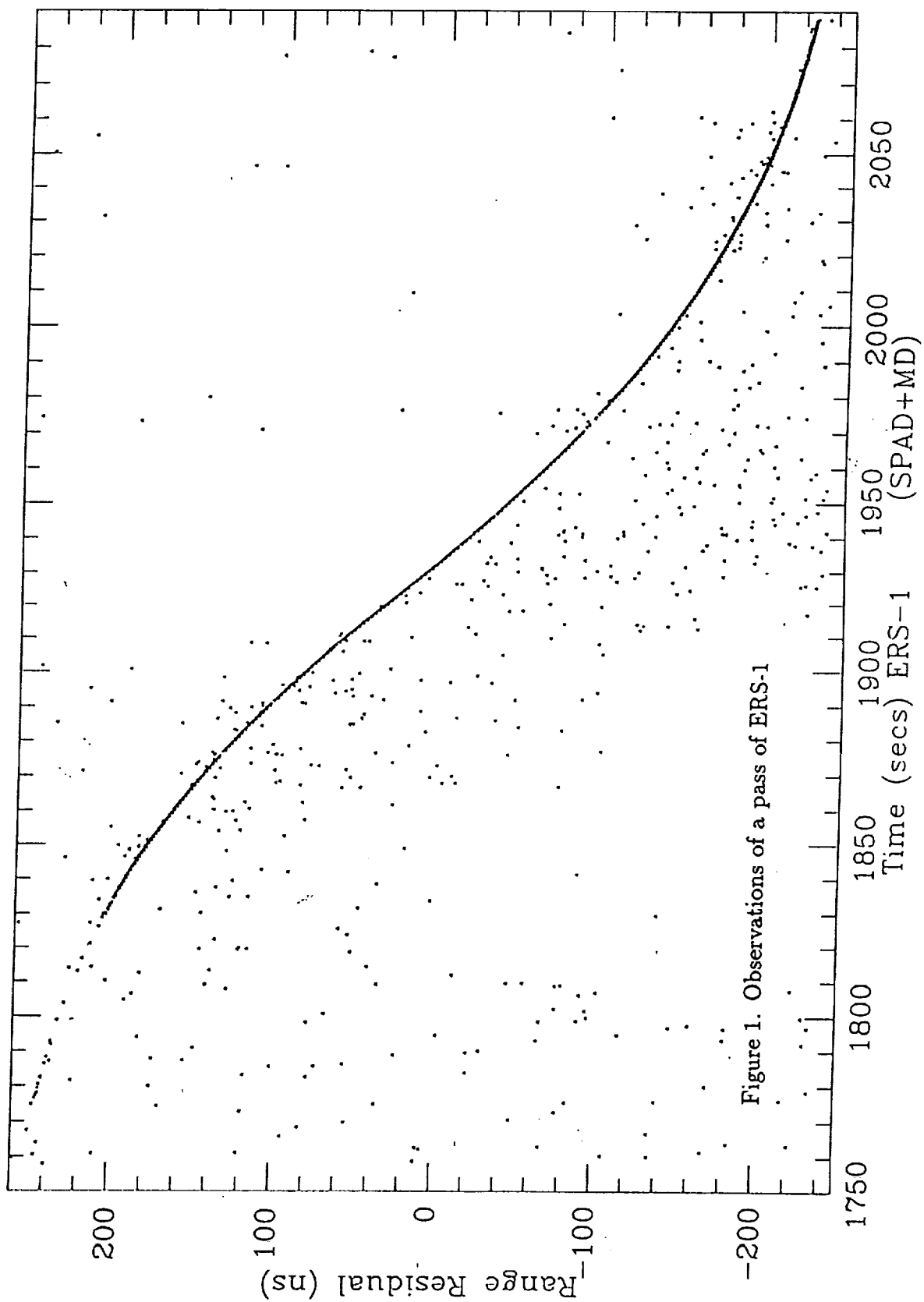
As a final check that the track of residuals is indeed statistically flat, we have introduced a single-factor analysis of variance test on the residuals. This test checks for significant differences between the means of residuals grouped into normal point bins; any differences indicate that not all trends have been removed from the residuals, and a warning is given that normal points should not be formed until the cause of the problem is traced. It is anticipated that this test may prove useful for the detection of system calibration changes during a pass.

3. Conclusion.

The scheme was found to be determinate for a wide range of satellites and pass durations and quality of predictions, and has been adopted at Herstmonceux. An earlier version of the software is also in use at several European SLR stations. The deduced values of time bias are used to good effect for improving subsequent predictions, and additionally the software has been used to generate time bias values from the quicklook observations from other SLR stations, as described by Wood and Gibbs (these proceedings).

4. References.

- Gaignebet, J. (Ed) 1985. Recommendation 84A, Normal Points, in *Proc. 5th Int. Workshop on Laser Ranging Instrumentation*. CERGA, France.
- Schutz, B.E. (Ed) 1990, April *Satellite Laser Ranging Newsletter*, Center for Space Research, University of Texas at Austin, USA.



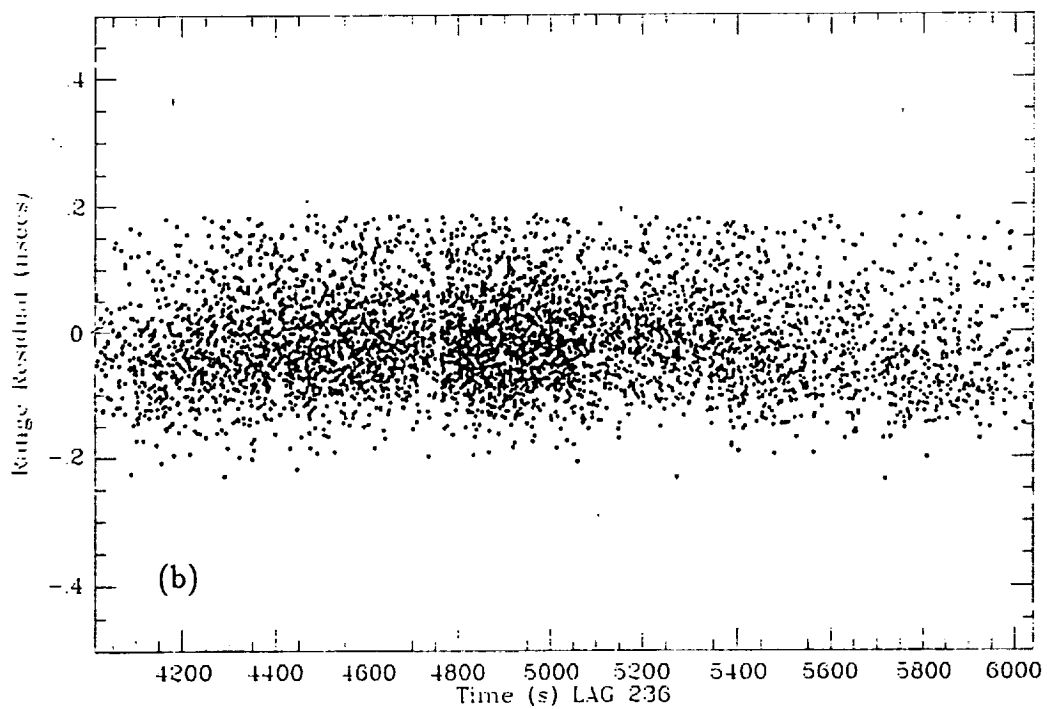
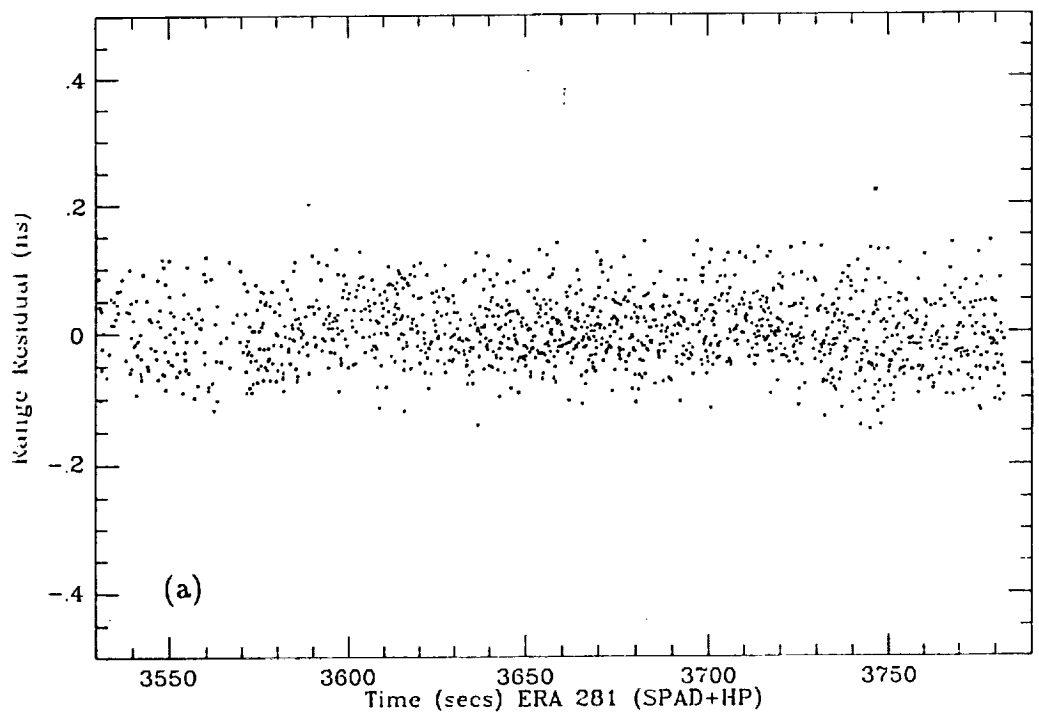


Figure 2. Range residuals from a pass of ERS-1 (a) and Lageos (b).